DeVoe
MAC 231
Calculus III

Test 2 A

Name DeVoe

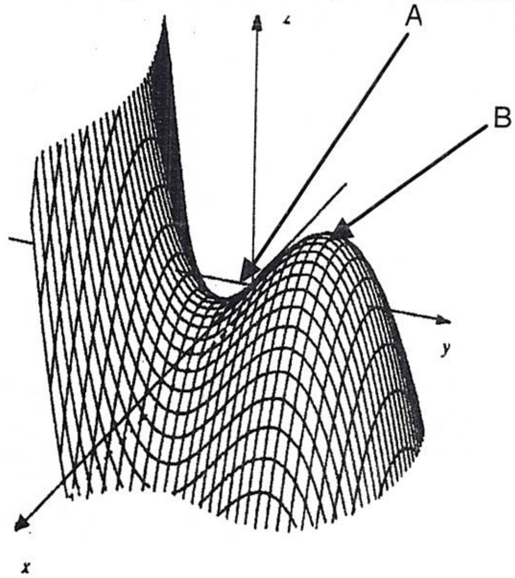
March 2017

NO CALCULATOR SECTION

For problems 1 – 4, place the CAPITAL letter of the best answer in the blank.

Refer to the diagram below for problems 1 – 3.

In the graph of f(x,y) shown below, A is a saddle point and B is a maximum.



1. A The value of f_{yy} at the point A in the graph shown above would be

concave up in y-direction

A) positive
B) zero
C) negative
D) no conclusion can be drawn

2. C Assuming $f_x = f_y = 0$ at point A in the graph shown above, the value of $D = f_{xx}f_{yy} - (f_{xy})^2$ would be saddle pt so D < 0

A) positive
B) zero
C) negative
D) no conclusion can be drawn

3. B Assuming the point B is a critical point, the value of $\vec{\nabla} f(B) =$ → $f_x = 0$ and $f_y = 0$

A) positive
B) $\vec{0}$
C) negative
D) no conclusion can be drawn

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4. ___B___ Suppose $f_x(x,y)$ is negative and $f_y(x,y)$ is positive for all (x,y). As (x,y) moves along the line from (2,-6) to (5,-6) , f(x,y) will

A) increase

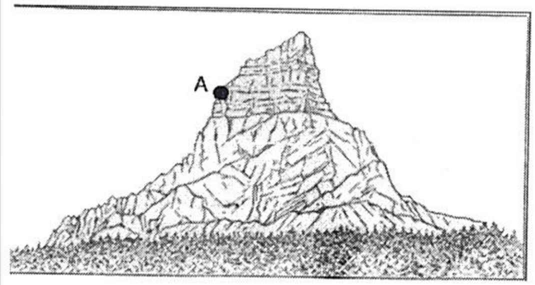
B) decrease

5. ___C___ Consider the sketch of a mountain shown. If you consider the surface of the mountain to be the function f(x,y), and the horizontal ground under the mountain to be the xy-plane, then at the point A, $\vec{\nabla} f$ points

A) up the mountain

B) down the mountain

C) parallel to the ground



For problems 5 – 6, show all your work and answers on THIS TEST PAPER.
Remember to simplify your answers!!

6. Given $f(a,b) = \frac{2a}{b^2} + a\sin(ab) + \ln(ab^2)$, find

A) $f_a(a,b)$

$f_a(a,b) = \frac{2}{b^2} + \sin(ab) + ab\cos(ab) + \frac{b^2}{ab^2}$

$\boxed{f_a(a,b) = \frac{2}{b^2} + \sin(ab) + ab\cos(ab) + \frac{1}{a}}$

B) $\frac{\partial f}{\partial b}(a,b) = -4ab^{-3} + a^2\cos(ab) + \frac{2ab}{ab^2}$

$\boxed{f_b(a,b) = -\frac{4a}{b^3} + a^2\cos(ab) + \frac{2}{b}}$

7. Let $f(x,y) = 3xy^3 + 2x^3y$, find

A) $\vec{\nabla} f(1,1)$

$\vec{\nabla} f = (3y^3 + 6x^2y)\hat{i} + (9xy^2 + 2x^3)\hat{j}$

$\vec{\nabla} f(1,1) = (3+6)\hat{i} + (9+2)\hat{j} = \boxed{9\hat{i} + 11\hat{j}}$

B) $f_{\vec{u}}(1,1)$ where $\vec{u} = \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$

$\vec{\nabla} f \cdot \vec{u} = \frac{9}{\sqrt{2}} - \frac{11}{\sqrt{2}} = \boxed{\frac{-2}{\sqrt{2}} \text{ or } -\sqrt{2}}$

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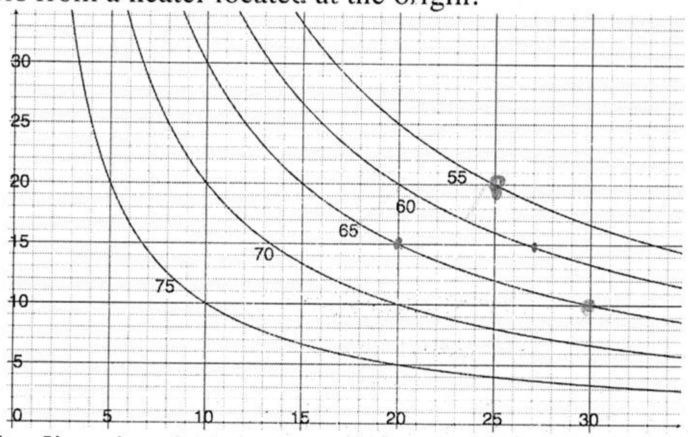
You may use a calculator on this part of the exam. Show all your work on your own paper, NOT ON THIS TEST PAPER. Do your work neatly and in order, labeling the A), B),...parts of a problem. Remember that you are graded on the work that is shown, so make sure it is accurate and complete. Use proper notation.

Refer to the diagram below to answer questions 1 – 3. This is a contour diagram of a function T(x,y) which gives the temperature in degrees Fahrenheit of a room at location x meters and y meters from a heater located at the origin.



1. **Did you read the directions?** Find the value of $T_x(20,15)$. Show the 3 – D coordinates of the points you use to calculate this value. Give the units. If the points you use are not given you will not receive full credit.

2. Which of the following vectors might represent the approximate direction of $\vec{\nabla} T(20,20)$? Choose from the list below and then explain your choice.
A) $\hat{i}+\hat{j}$ B) $-\hat{i}+\hat{j}$ C) $-\hat{i}-\hat{j}$ D) $\hat{i}-\hat{j}$

3. Which would be larger: $\vec{\nabla} T(25,20)$ or $\vec{\nabla} T(30,10)$? Explain. No calculations are needed.

4. Consider a pyramid with a square base of length x and height h. Suppose that the base and height depend on time. The volume, V, is given by $V=\frac{1}{3}x^2h$. At a certain instant the height is 5 cm and decreasing at the rate of 2 cm/sec and the base length is 1 cm and increasing at the rate of 3 cm/sec. What is the rate of change in volume at this instant? Include units.

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5. The temperature at points (x,y) on a rectangular plate is given by

$$T(x,y) = 96 - 5x^2 - 3y^2 - 3xy + 34x.$$

Find the coordinates of the hottest point on the plate. Use the 2nd derivative test to verify your answer.

6. The table below gives the results of a study done to measure the effect of exercise on the blood pressure of women. P = P(t, E) is the blood pressure, measured in millimeters of mercury (mm Hg), of women of age t years who are exercising at the rate of E watts.

TABLE 1. $P = P(t, E)$ (millimeters of mercury)

	$t = 25$	$t = 35$	$t = 45$	$t = 55$	$t = 65$
$E = 150$	178	180	197	209	195
$E = 100$	163	165	181	199	200
$E = 50$	145	149	167	177	181
$E = 0$	122	125	132	140	158

A) Use the table to estimate $P_E(65,0)$. Give units.

B) Interpret the meaning of the number you found in part A in the context of the problem.

C) Given that $P_t(65,0)$ is approximately 1.8 and your estimate from A, approximate the blood pressure for a woman who is 67 years old , who exercises at the rate of 8 watts. Explain/show your work. Answers without any supporting work will not be given credit.

7. Suppose that the depth, h, of a lake below the point (x,y) on the surface is given by $H(x,y) = 10xy - y^3 - 20$. Suppose a duck is at the point (3,2).

A) What will be the rate of change in the depth of the lake if the duck heads in the direction toward the point (7,6)? Include units.

B) In what direction should the duck swim in order for the depth to decrease most rapidly?

C) How fast will the depth decrease in this direction?

D) In what direction should the duck swim so that the depth remains the same?

8. Consider the function W = f(xy,z). The equation of the tangent plane to the level surface f(x,y,z) = 10 at the point (1,2,3) is $2x - 3y + 4z = 8$. Find $\vec{\nabla} W(1,2,3)$.

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1. $T_x(20,15) \approx \frac{\Delta T}{\Delta x} = \frac{65-60}{20-27} = \frac{5}{-7} = \boxed{-.714}$

$(20,15,65)$ and $(27,15,60)$

2. c) $-\hat{\imath} - \hat{\jmath}$ $\perp$ and pts in the direction of increase.

3. These are about the same $\Longrightarrow$ Question omitted

4. $V = \frac{1}{3}x^2 h$ $\quad \frac{dh}{dt}\Big|_{h=5} = -2$ $\quad \frac{dx}{dt}\Big|_{x=1} = 3$ $\quad \frac{dV}{dt} = ?$

$$\begin{array}{ccc} & V & \\ x & & h \\ | & & | \\ t & & t \end{array}$$

$\frac{dV}{dt} = V_x \frac{dx}{dt} + V_h \frac{dh}{dt}$ $\qquad V_x = \frac{2}{3}xh \qquad V_h = \frac{1}{3}x^2$

$V_x(5,1) = \frac{2}{3}(1)(5) = \frac{10}{3} \qquad V_h(5,1) = \frac{1}{3}(1)^2 = \frac{1}{3}$

$\frac{dV}{dt} = \left(\frac{10}{3}\right)(3) + \left(\frac{1}{3}\right)(-2)$

$= 10 - \frac{2}{3} = 9\frac{1}{3} = \boxed{\frac{28}{3}\ \frac{cm^3}{sec}}$

5. $T(x,y) = 96 - 5x^2 - 3y^2 - 3xy + 34x$

$T_x = -10x - 3y + 34 = 0 \qquad T_y = -6y - 3x = 0$

$-3x - 6y = 0$

$(-10x - 3y) = (-34)(-2)$

$-3x - 6y = 0$

$20x + 6y = 68$

$17x = 68$

$x = 4$

$-3(4) = 6y$

$-12 = 6y$

$-2 = y$

$\boxed{(4,-2)}$

$T_{xx} = -10 \qquad T_{yy} = -6 \qquad T_{xy} = -3$

$D = (-10)(-6) - (-3)^2 = 60 - 9 = 51 > 0 \Longrightarrow$ max or min

Since $T_{xx} = -10 < 0$ $\frown$ $\boxed{\text{max}}$

6. A) $P_E(65,0) \approx \frac{\Delta P}{\Delta E} = \frac{181-158}{50-0} = \frac{23}{50} = \boxed{.46}\ \frac{mg\ Hg}{watt}$

$P(\underset{t}{65}, \underset{E}{0}, 158)$ and $P(\underset{t}{65}, \underset{E}{50}, 181)$

B) A woman who is 65 years old, and is not exercising at all, will have her blood pressure increase by .46 mg Hg per watt of additional exercise.

c) $P(\overset{t}{67}, \overset{E}{8}) \approx ?$ $\quad \Delta P = 2(1.8) + 8(.46)$

$\Delta P = 7.28$

$P(65,0) = 158$

$P(67,8) \approx 158 + 7.28 = \boxed{165.28\ mmHg}$

7. $H(x,y) = 10xy - y^3 - 20$

A) From $(3,2)$ to $(7,6)$

$\vec{u} = \frac{4}{\sqrt{32}}\hat{i} + \frac{4}{\sqrt{32}}\hat{j} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

$\sqrt{32}$
$16 \cdot 2$

$H_{\vec{u}}(3,2) = ?$

$\vec{\nabla} H = 10y\,\hat{i} + (10x - 3y^2)\hat{j}$

$\vec{\nabla} H(3,2) = 20\hat{i} + (30 - \underset{12}{3 \cdot 4})\hat{j}$

$= 20\hat{i} + 18\hat{j}$

$\vec{\nabla} H_{\vec{u}}(3,2) = \frac{20}{\sqrt{2}} + \frac{18}{\sqrt{2}} = \boxed{\frac{38}{\sqrt{2}}}$

B) $\boxed{-\vec{\nabla} H = -20\hat{i} - 18\hat{j}}$

C) $\|\vec{\nabla} H\| = \sqrt{20^2 + 18^2} = \sqrt{724} \approx 26.9$

D) $\perp$ to $\vec{\nabla} H$ is tangent to the contour

In the direction $= \boxed{-18\hat{i} + 20\hat{j}}$

8. $2x - 3y + 4z = 8$

$\boxed{\vec{\nabla} W = 2\hat{i} - 3\hat{j} + 4\hat{k}}$