MAC 2313 Exam 1 Form A Name DeVoe

Calculus III DeVoe Spring 2017

Part I: NO CALCULATORS ARE ALLOWED ON THIS PART OF THE EXAM. Show all your work and answers on THIS test paper. Make sure your work is done neatly and is complete.

Please read the directions above! Use the vectors: $\vec{A} = \hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j} - \hat{k}$ to answer the following questions.

1. Find a vector 3 units long, in the **opposite** direction of $\vec{A}$. **3 points**

opposite $\vec{A} = -\hat{i} - 4\hat{j} + 3\hat{k}$

unit vector $= \dfrac{-\hat{i} - 4\hat{j} + 3\hat{k}}{\sqrt{1^2+4^2+3^2}} = -\frac{1}{\sqrt{26}}\hat{i} - \frac{4}{\sqrt{26}}\hat{j} + \frac{3}{\sqrt{26}}\hat{k}$

3 units long opp to $\vec{A}$ $= \dfrac{-3\hat{i} - 12\hat{j} + 9\hat{k}}{\sqrt{26}}$

2. Find: $\vec{A} \times \vec{B}$ **6 points**

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -3 \\ 3 & -2 & -1 \end{vmatrix} = -10\hat{i} - 8\hat{j} - 14\hat{k}$$

3. The result of $\vec{A} \times \vec{B}$ is a vector that is perpendicular to both $\vec{A}$ and $\vec{B}$. Show that this is true using your answer from number 2 and vector $\vec{A}$. **2 points**

$\vec{A} \cdot (\vec{A} \times \vec{B}) = (1 \times -10) + (4 \times -8) + (-3 \times -14) = -10 - 32 + 42 = 0$

$\vec{B} \cdot (A \times B) = (3 \times -10) + (-2 * -8) + (-1 * -14) = -30 + 16 + 14 = 0$

4. Write a sentence explain how you know this.(A picture will **not** be sufficient) **4 points**

MAC 2313
Calculus III

Exam 1 Form A

Name______________________

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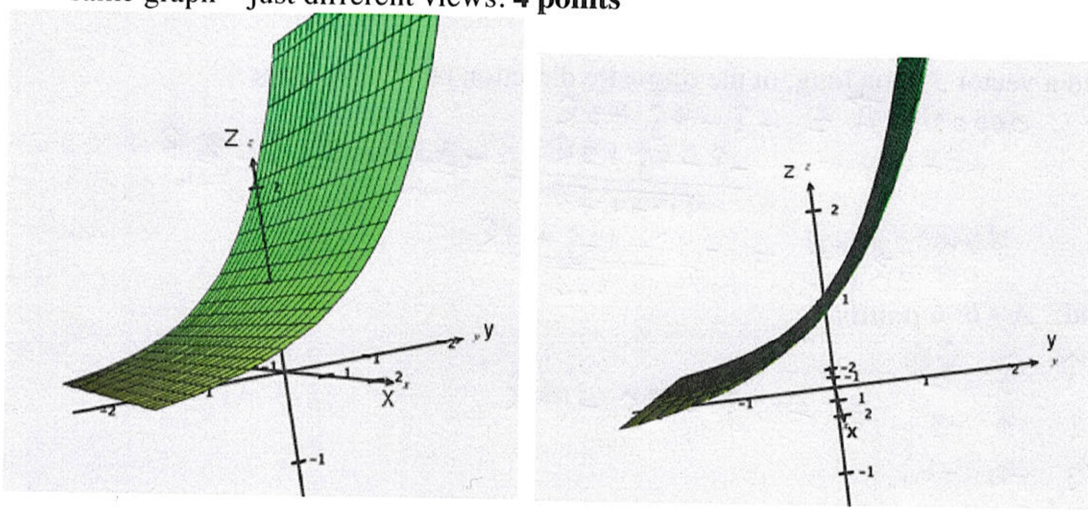
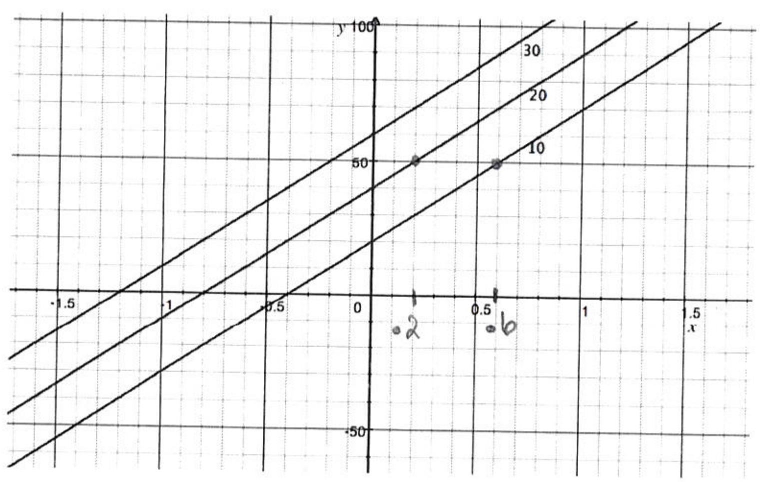
Part II: Show all your work and answers on your own blank paper, **NOT ON THIS TEST PAPER**. You must use pencil. Please do your work neatly and in order.

1. **Please read the directions above!** The sketch shown below is a cylinder. Give a possible equation of the exponential surface shown below. Both pictures are of the same graph – just different views. **4 points**



2. 2. Draw a contour diagram for the function $z(x,y)=\sqrt{x}-y$, by letting z = 0, 1, and 2. Draw and label each of 3 level curves , the axes, and show the equation used for each one. **10 points**

3. The contour diagram for a linear function is shown. Find the slope in the x –direction. Show ALL your calculations and the 3D coordinates of the points you used. **5 points**



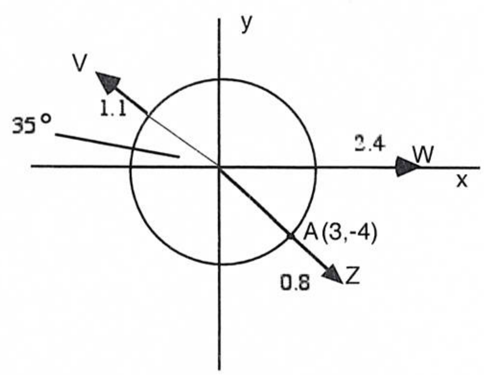
4. Airlines typically sell some tickets at full price and some at a discount. Suppose the revenue, R, in dollars from tickets sold on a particular flight, as a function of the number of full-price tickets sold, f, and the number of discount tickets sold, d is given by the function $R(f,d) = 239f + 79d$. Make sure you label part A and part B as two separate answers. To be clear, do not mix these 2 questions together. You should have an answer for A and an answer for B and each should be labeled.
   A) Give the units for the coefficient of f, which is 239 and the units on the coefficient of d, which is 79. **4 points**

   B) Write a sentence or two interpreting the meaning of those two coefficients in the context of the problem. **4 points**

5. True or False.
   A) True or False. Give your reasoning in either case. The value of $\vec{v} \times \vec{w}$ is never the same as $\vec{v} \cdot \vec{w}$, assuming the vectors are not 0. **4 points**

   B) True or False. Give your reasoning in either case. A normal(perpendicular) vector for the plane $z = 3x - 2y$ is $3\hat{i} - 2\hat{j}$ **4 points**

6. Three forces are acting on a ring as shown below where angles are measured in degrees and the magnitude of the forces are shown next to the vectors. To clarify, vector V makes an angle with the negative x-axis of 35° and has magnitude 1.1. Vector W lies along the positive x – axis with magnitude 2.4 and finally vector Z has magnitude 0.8 and passes through the point (3,-4). Find the sum of the vectors. Give your answer in **i, j** format and find the angle that the sum of the vectors makes with the positive x-axis. **8 points**



7. For the vectors **Z** = 4**i** - 3**j** + 10**k** and **V** = 3**i** + 5**j** + 2**k**
   A) Find the angle between **V** and **Z**. Give the answer to the nearest degree. **6 points**

   B) Find the vector component of **Z** parallel to, or in the direction of, **V**. Your answer should be written as a vector. **6 points**

8. A) Find the equation of a plane that contains the points P(-1,0,5) , Q(0,-2,3) and R(1,4,0). Full credit will only be given if you use vectors. **8 points**

   B) Find the equation of any plane that is parallel to the plane you found in A. There are many possible answers. **2 points**

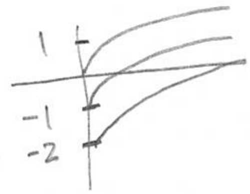
Calc3 EXAM 1 Form A Part II

1. $z = e^y$

2. $z = \sqrt{x} - y$ $\quad z=0 \quad y = \sqrt{x}$

$z=1 \quad y = \sqrt{x} - 1$

$z=2 \quad y = \sqrt{x} - 2$



3. $y=50 \quad x=.2 \quad x=.6$

$z=20 \quad z=10 \quad \text{slope} = \frac{10-20}{.6-.2} = \frac{-10}{.4} = -25 \text{ slope}$

4. A) $239 \frac{\$}{\text{full ticket}}$ And $79 \frac{\$}{\text{discount ticket}}$

B) \$239 is the price of a full-price ticket And \$79 is the price of a discount ticket

5. A) True because $\vec{v} \times \vec{w}$ is a vector $\vec{v} \cdot \vec{w}$ is a scalar. (3-D vector)

B) False. The normal vector is a multiple of $3\hat{\imath} - 2\hat{\jmath} - \hat{k}$

6. $\vec{V} = 1.1 \cos(145°)\hat{\imath} + 1.1 \sin(145°)\hat{\jmath}$

$\vec{W} = 2.4\hat{\imath} + 0\hat{\jmath} \qquad \text{Sum} = 1.98\hat{\imath} - .009\hat{\jmath} \qquad \measuredangle = \tan^{-1}\left(\frac{-.009}{1.98}\right)$

$\measuredangle = -.26°$

$\vec{Z} = .8\left(\frac{3}{5}\hat{\imath} - \frac{4}{5}\hat{\jmath}\right)$

7. $\vec{Z} = 4\hat{\imath} - 3\hat{\jmath} + 10\hat{k} \qquad \cos(\theta) = \frac{\vec{Z}\cdot\vec{V}}{\|\vec{Z}\|\,\|\vec{V}\|} = \frac{17}{\sqrt{125}\sqrt{38}} = .2467$

$\vec{V} = 3\hat{\imath} + 5\hat{\jmath} + 2\hat{k}$

A) $\underline{\theta = 75°}$

B) $\text{Proj}_V Z = (Z \cdot \hat{u}_v)\hat{u}_v = \frac{Z \cdot V}{\|V\|^2}\vec{V} = \frac{17}{38}(3\hat{\imath} + 5\hat{\jmath} + 2\hat{k})$

$= \frac{51}{38}\hat{\imath} + \frac{85}{38}\hat{\jmath} + \frac{34}{38}\hat{k}$

8. $\vec{PQ} = \langle 1, -2, -2 \rangle \quad \vec{PR} = \langle 2, 4, -5 \rangle$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & -2 \\ 2 & 4 & -5 \end{vmatrix} = 18\hat{\imath} - (-1)\hat{\jmath} + 8\hat{k}$$

$$= \underbrace{18\hat{\imath} + \hat{\jmath} + 8\hat{k}}_{\text{normal vector}}$$

$18(x+1) + (y-0) + 8(z-5) = 0$

$18x + y + 8z = 22$

$18x + y + 8z =$ Any number (except 22)

B) $18x + y + 8z$