Day 9
Derivatives of Functions of More Than One Variable

- Recall from Calculus I……
  - Limit Definition of the Derivative
    - $f'(a)=\lim\limits_{h\to 0}\frac{f(a+h)-f(a)}{h}$
    - $f'(a)=\lim\limits_{b\to a}\frac{f(b)-f(a)}{b-a}$
    - $f'(a)=\lim\limits_{\Delta x\to 0}\frac{\Delta f}{\Delta x}$
  - Find $f'(1)$ if $f(x)=ln(x)$, symbolically.
  - Estimate $f'(1)$ if $f(x)=ln(x)$
    - Estimate the Derivative Graphically.
    - Estimate the Derivative Numerically.
      - Using a table
      - Using Difference Quotients
- Now in Calculus 3, with more than one variable…..
  - Consider the table shown. It gives the temperature(°C) on a metal plate at a location (x,y) both given in meters. The function is T(x,y).
    - What is the temperature on the plate at the location (3,2)?
    - Is this a linear function?
    - T(x,y) can change in any direction we move from a given point and that change will not be constant.
    - Estimate the rate of change in the temperature T in the x – direction, holding y constant, as we move from the point (3,2) in the xy plane. This is called a *<u>partial derivative</u>* and the symbol is $T_x(3,2)$.
    - $\underset{\text{Hold } y \text{ constant at } 2}{T_x(3,2)}=\lim\limits_{\Delta x\to 0}\frac{\Delta T}{\Delta x}\approx\frac{\Delta T}{\Delta x}$, holding y constant at y = 2.
      Remember to stay <u>close</u> to the point (3,2).
    - Write a sentence interpreting what this number means in the context of the problem.

You Try It

Use the table to estimate $T_y(3,2)$. Also, write a sentence interpreting the meaning of the number you calculated in the context of the problem. Video Solution

Limit Definition of Partial Derivatives

- $T_x(a,b)=\lim\limits_{x\to a}\frac{T(x,b)-T(a,b)}{x-a}$ and $T_y(a,b)=\lim\limits_{y\to b}\frac{T(a,y)-T(a,b)}{y-b}$

- Leibnitz notation: $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial y}$.

- Consider the contour diagram shown. The function is H(x,t) where H is the temperature at a location x meters from a heater in a room, t minutes after it is turned on.
  - Use contour diagram to estimate H(20,30) and interpret in the context of the problem.
  - Use contour diagram to estimate H(10,35).
  - Use contour diagram to estimate $H_x(20,30)$ and interpret.

    $$\underset{\text{Hold } t \text{ constant at } 30}{H_x(20,30)} = \lim_{\Delta x \to 0} \frac{\Delta H}{\Delta x} \approx \frac{\Delta H}{\Delta x}$$

You Try It

Use the contour diagram to estimate $H_t(20,30)$. Also, write a sentence interpreting the meaning of the number you calculated in the context of the problem. Video Solution

- What's going on graphically?
- Partial Derivatives Symbolically
  - Given $f(x,y) = x^3y^4 + y^5 + e^{-2x}$ find each partial derivative.
  - Given $z = y^2e^{-xy}$ find each partial derivative.
  - Given $a = \frac{v^2}{r}$ find each partial derivative.

You Try It

- Try Section 14.2 #11 Answer in the back of the text.
- Given $z = \frac{x}{x+y}$ find each partial derivative. Video Solution
- Given $w = \ln(x^2 + y^2)$ find each partial derivative. Video Solution