Day 5

Applications of the Dot Product

- Vector Projections
  - Can we break **v** into components that are || and $\perp$ to **w**?
    - $\vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp}$
  - How much of **v** goes in the direction of **w**?
  - If there were a light above **v**, what shadow would **v** cast on **w**?
  - Procedure
    - Show the length of $\vec{v}_{\parallel} = \|\vec{v}\|\cos(\theta)$
    - Show that $\vec{u} \cdot \vec{v} = \|\vec{v}\|\cos(\theta)$, where **u** is a unit vector in the direction of **w**.
    - So $\vec{u} \cdot \vec{v}$ gives the scalar amount of **v** in the direction of **u** or the length of $\vec{v}_{\parallel}$.
    - The <u>vector</u> we want has length given by $\vec{u} \cdot \vec{v}$ and the direction of **u**.
    - So $\vec{v}_{\parallel} = (\vec{u} \cdot \vec{v})\vec{u}$, where the $(\vec{u} \cdot \vec{v})$ gives the length of the vector we're looking for and the unit vector **u** gives it the correct direction.
    - To find $\vec{v}_{\perp}$ note that $\vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp}$ so if we solve this equation for $\vec{v}_{\perp}$ we get $\vec{v}_{\perp} = \vec{v} - \vec{v}_{\parallel}$.
  - Example:
    - Find the vector projection of $\vec{A} = \hat{i} + 4\hat{j}$ on to $\vec{B} = 3\hat{i} + 4\hat{j}$.

      NOTE: Another way to say this is: write **A** as the sum of vectors || and $\perp$ to **B**

  You Try It

  Section 13.3 #46 Answer: $\vec{F}_{\parallel} = \frac{-29}{5}\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right)$

- Equations of Planes using vectors
  - Find the equation of a plane which is $\perp$ to $\vec{v} = \hat{i} + 2\hat{j} + 3\hat{k}$ and passes through the point (4,5,6)
    - Note that this similar to lines in 2D: in 2D you are given a point and the "direction" of the line given by the slope. In 3D, you are given a point and the "direction" of the plane given by the perpendicular vector.
  - Procedure
    - Let (x,y,z) be any point in the plane.
    - Form the vector from the point (4,5,6) to the point (x,y,z).
      - Final – Initial: $\vec{A} = (x-4)\hat{i} + (y-5)\hat{j} + (z-6)\hat{k}$
    - Since these points (4,5,6) and (x,y,x) are both in the plane the vector we formed **A,** is also in the plane.
    - The given vector $\vec{v} = \hat{i} + 2\hat{j} + 3\hat{k}$ is $\perp$ to the plane and therefore $\perp$ to the vector **A.**

- The dot product between these two vectors must be 0.
  - This gives us x + 2y = 3z = 32.
  - We DO NOT need to go through this whole procedure every time.
  - Note that the coefficients of the variables in the plane equation are the coefficients of the i, j, and k vectors in the given plane. This is always true.
  - So we can short cut the process by just writing down the left hand side of the plane equation = D and then use the given point to find D.
- In general: If $\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$ is a vector perpendicular to a plane then the plane equation is given by ax + by + cz = D.

You Try It

Section 13.3 # 13 Answer in Text

You Try It

Section 13.3 #15 Answer in Text