Day 28

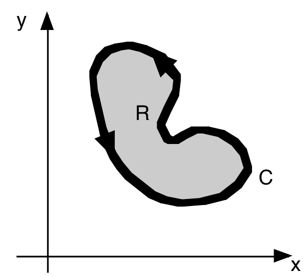
Green's Theorem

- If the vector field is path independent, then the line integral around a closed path, called the circulation, is 0.
    - Calculate $\oint_C \vec{F} \cdot d\vec{r}$ for $\vec{F} = (6x + 16xy)\hat{i} + (8x^2 + 18y)\hat{j}$ around the unit circle.
- But what if the vector field is not path independent and the path is closed?

Green's Theorem

If R is a simple region in the plane whose boundary is the simple closed path C, oriented in the counterclockwise direction, and $\vec{F}(x,y) = f(x,y)\vec{i} + g(x,y)\vec{j}$ **then:*

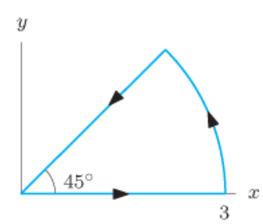
$$\int_C \vec{F} \cdot d\vec{r} = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA$$



*f and g must have continuous first partials on R

- Say WHAT?
- Apply Green's Theorem to above example to illustrate.
- Example 1: Calculate the circulation of $\vec{F}$ around the unit circle, oriented counterclockwise, where $\vec{F} = y\hat{i} - x\hat{j}$, two ways:
    - Green's Theorem
    - Definition of the line integral

- Example 2: Find the line integral of $\vec{F} = (x - y)\hat{i} + x\hat{j}$ around the closed curve shown which is 1/8 of a circle with radius 3.



You Try It

Section 18.4 #21. Answer in the text.

- Example 3: Find $\oint_C \vec{F} \cdot d\vec{r}$ for $\vec{F} = -x^2 y\hat{i} + xy^2\hat{j}$ where C is the unit square centered at the origin, traversed counterclockwise.

- **Summary** First check to see if $\vec{F}$ is path independent. If it is, find f(x,y) and use it and the First Fundamental Theorem of Line Integrals to evaluate the line integral. This way you just need the end points – not the path.

- And, bonus, if it is path independent and the path is closed, then the line integral around the closed path is 0.
- If the vector field is not path independent then you must use the parameterization of the path to write $\vec{F}$ in terms of *t* and calculate the dot product of that and $\frac{d\vec{r}}{dt}$ to evaluate the line integral.
- If the vector field is not path independent and the path is closed, use Green's theorem. If $\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}$ is a constant, think about the geometry of the region enclosed by the path – you might catch a break and be able to use a geometry formula for the area of the region.