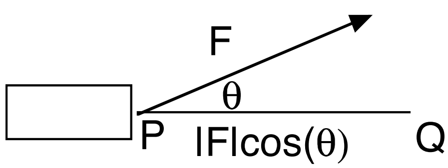
Day 25
Vector Fields and Line Integrals

- Vector Field is a vector function that assigns a vector to each point in space.
  - Velocity Fields
    - Fluid Flow
    - Wind Flow
  - Force Fields
    - Gravitational
    - Electromagnetic
  - Example 1: $\vec{F}(x,y)=(x-2y)\hat{i}+(x+y)\hat{j}$
    - By Hand
    - On Computer
  - Example 2: For the following think it out, then draw by hand, then go to the computer to check.
    - Draw the vector field given by $\vec{F}(x,y)=y\hat{i}$

You Try it

Draw the vector field given by $\vec{G}(x,y)=y\hat{j}$ Check on the computer.

  - Example 3: Gradient Vector Fields
    - Let $f(x,y)=5-x^2-y^2$. Then the gradient vector $\vec{\nabla}f=-2x\hat{i}-2y\hat{j}$ is a vector field.
- Line Integrals
  - Measures the extent to which a curve in a vector field is, overall, going with the vector field or against it.
  - Recall: Work = Force ×Distance
    - If $\vec{F}$ is in the direction of motion, then Work = Force ×Distance
    - But if



then Work = Force in the direction of motion×Distance

$$Work=\|\vec{F}\|\cos(\theta)\|\overline{PQ}\|$$
$$=\|\vec{F}\|\|\overline{PQ}\|\cos(\theta)$$
$$=\vec{F}\cdot\overline{PQ}$$

- Now, suppose we have a vector field and an oriented curve.....
  - $W=\sum_{i=1}^{n}\vec{F}\cdot\Delta\vec{r}_i$
  - Let $\|\Delta\vec{r}\|\to 0$ or $n\to\infty$
  - $W=\lim_{\|\Delta\vec{r}\|\to 0}\sum_{i=1}^{n}\vec{F}\cdot\Delta\vec{r}_i=\int_C\vec{F}\cdot d\vec{r}$: Work done by $\vec{F}$ along C.

- Interpreting $\int_C \vec{F}\cdot d\vec{r}$
  - In general, the line integral of a vector field $\vec{F}$ along a curve C measures the extent to which C is going with $\vec{F}$ or against it.
    - The line integral is 0 if $\vec{F}$ is perpendicular to the path at all points or if the positive and negative contributions cancel each other out.
    - If $\vec{F}$ is mostly pointing in the same direction as C then the line integral is positive and it will be negative if they are acting in opposite directions. However, care must be given to also consider the magnitude of $\vec{F}$.
  - Example 4: Arrange the line integrals $\int_{C_i} \vec{F}\cdot d\vec{r},\ for\ i=1,2,3,4$ in ascending order.

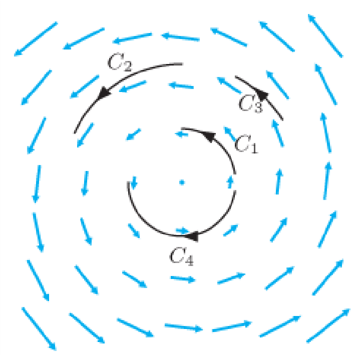


Figure 18.5 Vector field and paths C_1, C_2, C_3, C_4

You Try It

Section 18.1 #30 Answer: C_2.

- Calculating a "basic" $\int_C \vec{F}\cdot d\vec{r}$
  - Example 5: Calculate the line integral: $\int_C \vec{F}\cdot d\vec{r}$ if $\vec{F}=y\hat{i}$, from (0,1) to (0,5).
  - Example 6: Calculate the line integral: $\int_C \vec{F}\cdot d\vec{r}$ if $\vec{F}=6\hat{i}+y^2\hat{j}$, from (3,0) to (7,0).

You Try It

Section 18.1 #7 Answer: 0.

- If C is a closed curve then $\int_C \vec{F}\cdot d\vec{r}$ is called the ***circulation*** of $\vec{F}$ around C.

Wrap Up