Day 22
Position, Velocity and Acceleration vectors
- [Position vectors and parameterized curves]
  - $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$
  - For example, the parametric equations for the circle, $\begin{array}{l} x(t) = \cos(t) \\ y(t) = \sin(t) \end{array}$, can

    be written with the position vector $\vec{r}(t) = \cos(t)\hat{i} + \sin(t)\hat{j}$. The tip of the vector traces out the circle as t changes.
  - [Using the 3D Plotter]
  - [Helix]:

    $x(t) = 3\cos(t)$
    $y(t) = 3\sin(t)$ $\vec{r}(t) = 3\cos(t)\hat{i} + 3\sin(t)\hat{j} + t\hat{k}$
    $z(t) = t$

- Velocity, Speed and Acceleration
  - [Example 1]: $\vec{r}(t) = 30t\,\hat{i} + (48t - 16t^2)\hat{j}$
    - Find the velocity vector $\vec{v}(t)$ and $\vec{v}(2)$
    - Find the speed, $\|\vec{v}(t)\|$ and $\|\vec{v}(2)\|$.
    - Velocity is tangent to the curve in the direction of motion.
    - Find the acceleration vector, $\vec{a}(t)$.

  - In general,
    Position: $\vec{r}(t)$ Traces out the motion of the particle.
    Velocity: $\vec{v}(t) = \vec{r}\,'(t)$ Tangent to the curve in the direction of motion.
    Speed: $\|\vec{v}(t)\|$ Magnitude of the velocity
    Acceleration: $\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$
  - [Helix: Velocity, Speed, Acceleration]

    🖊 *You Try It*
        Do Section 17.2 # 1,7 Answer in Text

- Uniform Circular Motion
  - [Example 2:] For $\vec{r}(t) = 2\cos(\pi t)\,\hat{i} + 2\sin(\pi t)\hat{j}$, $0 \le t \le 2$
    - Find $\vec{r}(1)$, $\vec{v}(1)$ and $\vec{a}(1)$.
    - Is $\vec{a}(1) \perp \vec{v}(1)$?
    - Is $\vec{a}(t) \perp \vec{v}(t)$ for all t?

    🖊 *You Try It*
        Do Section 17.2 # 17 Answer in Text

- A child wanders down a circular staircase from the top of a tower. With *x, y* and *z* in feet and the origin at the base of the tower, her position from the start is given by

$$x(t) = 10\cos(t)$$

$$y(t) = 10\sin(t)$$

$$z(t) = 90 - 5t$$

  a) How tall is the tower?
  b) When does the child reach the bottom?
  c) What is her speed at time *t*? At *t = π*?
  d) What is her acceleration at time *t*?

<div align="right">From: Calculus Hughes- Hallet, et al.</div>