Day 21
Integrating in Spherical Coordinates and Parametric Curves

- Integrating in Spherical Coordinates
  - What does ΔV look like?
  - $\iiint_R f(\rho,\theta,\phi)\ \Delta V = \iiint_R f(\rho,\theta,\phi)\ \rho^2 \sin(\phi)\ d\rho\ d\phi\ d\theta$
  - Example 1: Find the volume of a sphere of radius R

    You Try It
    Do Section 16.5 # 15 Answer in Text

  - Example 2: Find the mass of the solid region W given in spherical coordinates by $0 \le \rho \le 3,\ 0 \le \theta \le 2\pi$ and $0 \le \phi \le \frac{\pi}{4}$. The density at any point P is the distance of P from the origin.

    You Try It
    Do Section 16.5 # 25 Answer in Text

  - Example 3: Write $\iiint_S \left(x^2+y^2+z^2\right)^{3/2}\ dz\ dy\ dx$ in spherical coordinates where S is a sphere of radius a.

    You Try It
    Do Section 16.5 # 24 Video Solution

Parameterizing Curves

- What is a Parametric Equation?
  - $x(t) = t$
    $y(t) = t^2$
  - Parametric equations on your calculator
    - Shifting: vertically and horizontally
    - Switch $x(t)$ and $y(t)$
  - Any function, $f(x)$, can be parameterized by using: $x(t) = t$, $y(t) = f(t)$
  - Important to state the domain of the parameter, *t*.

    You Try It
    Do Section 17.1 # 1 Answer in Text

- Parameterizing Circles
  - $x(t) = 2\cos(t)$
    $y(t) = 2\sin(t)$
  - t – step and t interval
  - Direction, starting and ending points
  - Center, radius
  - Ellipse
  - Speed
    - $x(t) = \cos(t)$ $y(t) = \sin(t)$ and $x(t) = \cos(2t)$ $y(t) = \sin(2t)$

    You Try It

    Do Section 17.1 # 2 and #21(you can always – at least for 2D - check that you have the correct equations by using your calculator) Answer in Text
    Do Section 17.1 # 33 Answer in Text

- In 3D:
  - What is?
    - $x(t) = 4\cos(t)$
      $y(t) = 0$
      $z(t) = 4\sin(t)$
  - What is?
    - $x(t) = \cos(t)$
      $y(t) = \sin(t)$
      $z(t) = t$

    You Try It

    Do Section 17.1 # 19 Answer in Text
    Do Section 17.1 # 39 Answer in Text

- Lines in 3D
  - Find parametric equations for a line that is parallel to the vector $\vec{v}(t) = \hat{i} + 2\hat{j} + 3\hat{k}$ and passes through the point (4,5,6)
  - Lines in general:

If a line passes through the point (x_0, y_0, z_0) and is parallel to the vector $\vec{v}(t) = a\hat{i} + b\hat{j} + c\hat{k}$, then the parametric equations that describe this line are given by:

$$x(t) = x_0 + at$$
$$y(t) = y_0 + bt$$
$$z(t) = z_0 + ct$$

You Try It

Do Section 17.1 # 11 Answer in Text
Do Section 17.1 # 13 Answer in Text