

Day 20

Triple Integration using Cylindrical Coordinates and Intro to Spherical Coordinates

- Cylindrical Coordinates

- Polar in 3D – just add the z coordinate on.
- r is measured as in Polar except $0 \leq r < \infty$
- θ is measured as in Polar except: $0 \leq \theta \leq 2\pi$
- z is measured as in rectangular: $-\infty < z < \infty$

- Plot a point $\left(2, \frac{\pi}{4}, 3\right)$

- Describe the equations in Cylindrical Coordinates

- $r = 2$
- $\theta = \frac{\pi}{4}$
- $z = 3$

- Integrating in Cylindrical Coordinates

- What does ΔV look like?

- $\int_R^R \int_0^{\pi} \int_0^h f(r, \theta, z) r \, d\theta \, dr \, dz$

- Note: If $f(r, \theta, z) = 1$, then

$$\int_R^R \int_0^{\pi} \int_0^h 1 \, d\theta \, dr \, dz = \int_R^R 1 \, dr \, dz = \text{Volume of } R$$

- Example 1: Find the volume of a cylinder with Radius R and height H . (This is included in the video above.)

 *You Try It*

Do Section 16.5 #13 Answer in Text

- Example 2: Find the volume between the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 4$

 *You Try It*

Do Section 16.5 # 33 Answer in Text

- Spherical Coordinates

- ρ is the distance measured out from the origin to the point, $0 \leq \rho < \infty$
- ϕ is the angle measured down from the positive z – axis, $0 \leq \phi \leq \pi$
- θ is the same θ from Polar coordinates, $0 \leq \theta \leq 2\pi$
- Here's a link to a site that will help you get comfy with Spherical Coordinates

- Plot a point $\left(3, \frac{\pi}{4}, \frac{\pi}{2}\right)$, given as (ρ, θ, ϕ)

- Describe the equations in Spherical Coordinates

- $\rho = 2$
- $\theta = \frac{\pi}{4}$
- $\phi = \frac{\pi}{4}$
- $\phi = \frac{3\pi}{4}$
- $\phi = \frac{\pi}{2}$ and $\rho = 2$
- $\phi = \frac{\pi}{2}$ and $0 \leq \rho \leq 2$

▪ [Practice Problems on Spherical Coordinates Website](#)

- [Conversion Equations](#)