Day 19
Double Integration in Polar Coordinates

- Brief Review of Polar Coordinates
  - Locating Points
  - Conversion Equations
  - Describe the graph of
    - $r = 4$
    - $\theta = \frac{\pi}{4}$
- Why is integration in Polar sometimes preferred?
  - Double Integration in Polar Coordinates
  - In the rectangular coordinate system, when integrating over a region in the xy-plane, each ΔA "element" was a rectangle with area $\Delta A = \Delta x\ \Delta y$.
  - In polar coordinates the ΔA "element" is a "polar" rectangle and the area is $\Delta A = r * \Delta\theta\ \Delta r$.
  - $$\sum f \cdot \Delta A = \sum f(r,\theta) \cdot r\ \Delta\theta\ \Delta r$$
  - $$\lim_{\Delta\theta,\Delta r \to 0} \sum f(r,\theta) \cdot r\Delta\theta\ \Delta r = \iint_R f(r,\theta)\ r\ d\theta\ dr$$
  - Note: If $f(r,\theta)$=1 then,
    $$\iint_R f(r,\theta)\ r\ d\theta\ dr = \iint_R 1\ r\ d\theta\ dr = Area\ of\ R$$
  - Note: If $f(r,\theta)$= density of R then,
    $$\iint_R f(r,\theta)\ r\ d\theta\ dr = Mass\ of\ R$$
  - Example 1: Find the volume of the solid bounded below by the xy-plane and above by $z = 25 - x^2 - y^2$.

    You Try It
    Do Section 16.4 #17 Answer in text.
  - Example 2: Sketch the region of integration: $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^1 f(r,\theta)\ r\ dr\ d\theta$

    You Try It
    Do Section 16.4 #9 Answer in text.

- Example 3:Convert the following integral to Polar Integration

$$\int_{-1}^{0}\int_{0}^{\sqrt{1-x^2}} x\ dy\ dx$$

You Try It

Do Section 16.4 #19 Answer in text.

- Example 4: Suppose a circular plate, centered at the origin with radius 5 has density, $\delta(x,y)=xy$. Find the mass of the quarter of the circular plate located in the first quadrant.