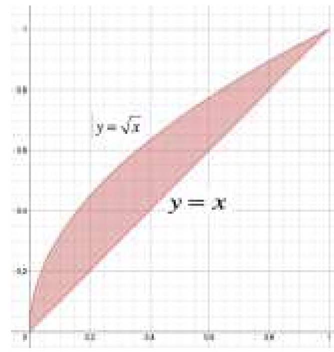
Day 18

Applications and Triple Integration

- Mass in 2D
  - Example 1: Find the mass of the "thin" (VERY thin) plate shown in the figure if the density $\delta(x,y)=x^2y\ \frac{kg}{m^2}$



- Mass in 3D
  - Approximate the mass of the object given:

Color	Density gm/in^3
Red	2
Blue	3
Yellow	4
Green	6
Orange	7

  - Mass = Σ Density •Volume

  - What if the density is given as a function of x, y, and z: $\delta(x,y,z)$?
    - Mass of each block = $\delta\ \Delta V$
    - Mass of entire region R $\approx \sum \delta\ \Delta V$

      $$Mass = \lim_{\Delta V\to 0}\sum \delta\ \Delta V = \iiint_R \delta\ dV$$

    - Example 2: Suppose a rectangular block has density given by $\delta(x,y,z)=1+xyz$ and the block is oriented so that the 2" side is along the x –axis, the 3" side is along the y – axis and the 5" side is along the z – axis. Find the mass of the block. (Does it matter how it is oriented?)
  - So the mass of a 3D object covering the region in 3 space given by R with density $\delta(x,y,z)$ is given by:

    $$Mass = \iiint_R \delta(x,y,z)\ dV$$

  - Note: What if $\delta = 1$?

    $$Volume = \iiint_R dV$$

- Example 3 Problem 40 Section 16.3
- Example 4 Problem 42 Section 16.3

You Try It

Do Section 16.3 # 39 Answer in text.

You Try It

Do Section 16.3 # 41 Answer in text.

- Example 5 Sketch the region of integration for $\int_0^1 \int_{-1}^1 \int_0^{\sqrt{1-x^2}} f(x,y,z)\ dz\ dx\ dy$
- Example 6 Sketch the region of integration for
$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} f(x,y,z)\ dz\ dx\ dy$

You Try It

Do Section 16.3 #7 Answer in text.

You Try It

Do Section 16.3 #9 Answer in text.

- Example 7 Write a triple integral, including limits of integration, that gives the volume between $z = x^2 + y^2$ and $x^2 + y^2 + z^2 = 4$ and above the disk in the xy-plane $x^2 + y^2 \le 1$. You do not need to integrate it – you're welcome!

You Try It

Do Section 16.3 #37 Answer in text.