Day 17

Unit 3 Intro

Double Integration

- Recall From Calc I…
  - Estimating area under the curve
  - $\lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i)\ \Delta x = \int_a^b f(x)\ dx$
- In Calc III…..
  - Developer wants to buy a parcel of land. How much does the parcel cost?
    - Constant price/yd^2
    - Non-constant price/yd^2
    - Contour Diagram
    - On average, what is the price per acre? Note: 4840 yd^2 = 1 acre
  - $\lim_{\Delta x, \Delta y \to 0} \sum f(x_i, y_j)\ \Delta x\ \Delta y = \int_R f(x,y)\ dA$
  - If *x,y,* and *z* are lengths, then the volume under $f(x,y)$ and above *R* is $V = \int_R f(x,y)\ dA$.
  - The average value of $f(x,y)$, no matter what *f* stands for is

$$f_{average} = \frac{1}{Area\ of\ R} \int_R f(x,y)\ dA$$

  - Example: Give a lower estimate for $\int_R f(x,y)\ dA$ where $f(x,y) = 100 - x^2 - y^2$ and $R: 1 \le x \le 3,\ 0 \le y \le 6$. Use $\Delta x = 1$ and $\Delta y = 2$.

You Try It

Give an Upper Approximation for the problem above. Answer: 1096

  - How do we calculate the integral exactly?
    - Revisit the land problem
    - $\lim_{\Delta x, \Delta y \to 0} \sum f(x_i, y_j)\ \Delta x\ \Delta y = \iint_R f(x,y)\ dA$

$$\int_{y=c}^{y=d} \int_{x=a}^{x=b} f(x,y)\ dx\ dy = \int_{x=a}^{x=b} \int_{y=c}^{y=d} f(x,y)\ dy\ dx,$$

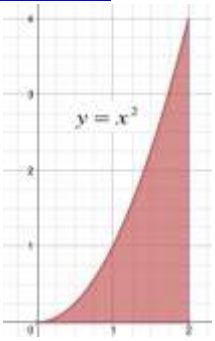
where $R: a < x < b,\ c < y < d$

  - Example: Find the volume under $f(x,y) = 2 + xy$ and above the region in the xy-plane given by: $0 < x < 2,\ 0 < y < 4$. (This is example is shown in video above.)
    - Integrating with respect to x first.
    - Integrating with respect to y first.

You Try It

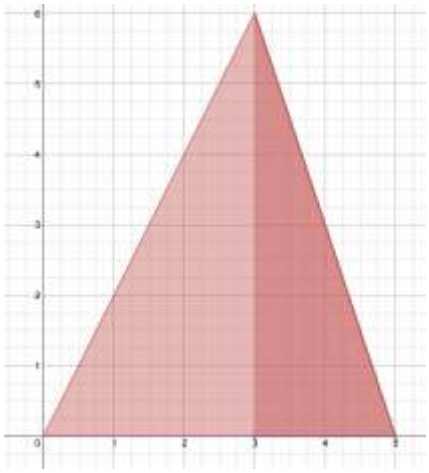
1. Evaluate $\int_1^3 \int_0^2 5 + x^2 + y^2 \; dy \; dx$
2. Switch the order of integration to confirm you get the same answer either way. Video Solution to 1 and 2
3. Interpret your answer as a volume. Solution Part 3
4. If $f(x,y) = 5 + x^2 + y^2$ represented the temperature of a metal plate in the xy – plane such that $1 < x < 3,\ 0 < y < 2$, find the average temperature of the plate over this region. Solution Part 4

- What if the region of integration is not rectangular?
  - Example 1: Evaluate $\iint_R 3xy^3 \; dA$ where R is the region shown in the graph.
    - dy dx
    - dx dy



  - Example 2: Switch the order of integration and integrate. $\int_0^1 \int_y^1 \sin(x^2) \; dx \; dy$.

  - Example 3: Write $\iint_R f \; dA$ as an iterated integral for the shaded region R.



You Try It

Section 16.2 #21 and Section 16.2 #33. Answer in the text.