Day 13
Directional Derivatives of Functions of Three Variables

- Everything we said about directional derivatives of functions of 2 variables, $f(x,y)$ carries over to functions of 3 variables, $f(x,y,z)$.
    - $\vec{\nabla} f = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$ and $\vec{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$
    - $f_{\vec{u}}(a,b,c) = \vec{\nabla} f \cdot \vec{u} = f_x(a,b,c)u_1 + f_y(a,b,c)u_2 + f_z(a,b,c)u_3$
    - Example: Given $f(x,y,z) = x^2 + y^2 + z^2$, find $f_{\vec{u}}$ at the point (1,2,3) in the direction of the point (1,1,1).

You Try It

Section 14. 5 #19 Answer in text

- Properties of $\vec{\nabla} f$ in 3 space are similar.
    - $\vec{\nabla} f$ points in the direction of Greatest Rate of Increase in *f*
    - $-\vec{\nabla} f$ points in the direction of Greatest Rate of Decrease in *f*
    - $\left\|\vec{\nabla} f\right\|$ gives the greatest rate of increase.
    - $\vec{\nabla} f$ is perpendicular to the level surface of *f* at the point *(a,b,c)*
- Finding Tangent planes using $\vec{\nabla} f$.
    - By thinking of $x^2 + y^2 + z^2 = 14$ as a level surface of the function $f(x,y,z) = x^2 + y^2 + z^2$, find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 = 14$ at the point (1,2,3).

You Try It

Section 14.5 #39 Answer in text.

Higher Order Partials

- You can continue to take derivatives to find 2nd order derivatives and higher.
    - Example 1: Given $f(x,y) = x^2y^4 + 5x^2 + e^{2y}$, find the first and second order partial derivatives. Also show some higher order derivatives.
    - Example 2: Given $f(x,y) = xe^{xy}$, find the first and second order partial derivatives.

You Try It

Section 14.7 #5 Answer in text.

- Recall from Calculus I that the second derivative tells you how fast the first derivative is changing. Graphically, this means it gives you information where the function is concave up, $f'' > 0$, and where the function is concave down, $f'' < 0$.
- The same is true in Calculus III
    - $f_{xx} > 0$: concave up in the x – direction.
    - $f_{yy} > 0$: concave up in the y – direction.
    - $f_{xx} < 0$: concave down in the x – direction.

- $f_{yy} < 0$: concave down in the y – direction.
    - Example: Consider the function $f(x, y) = x^2 - y^2$.
      Is $f_{xx}(0,0)$ positive, negative or 0?
      Is $f_{yy}(0,0)$ positive, negative or 0?

The Chain Rule for functions of many variables

- Recall from Calculus I……
    - If *y* is a function of *x*, or *y(x)*, and *x* is a function of *t*, or *x(t)*, then $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$.
    - We only need one chain rule since y can only be a function of one variable.
- In Calculus III……
    - We need a different chain rule for each situation depending on the different combination of variables.
    - Example: Suppose $z(x, y)$, $x(t)$ and $y(t)$.
        a. Write the chain rule for $\frac{dz}{dt}$ by making a tree diagram.
        b. Apply the chain rule you found in part a for the functions: $z(x, y) = x^2 + y^2$, $x(t) = \sin(2t)$ and $y(t) = \cos(2t)$.
    - Example: Suppose $z(x, y)$, $x(r, \theta)$ and $y(r, \theta)$.
        a. Write the chain rule for $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ by making a tree diagram.
        b. Apply the chain rule you found in part a for the functions: $z(x, y) = x^2 + y^2$, $x(r, \theta) = r\cos(\theta)$ and $y(r, \theta) = r\sin(\theta)$.

You Try It

Section 14.6 #5

Section 14.6 #9 Answers in Text.