Day 12
More on the Directional Derivative

- Recall: Directional derivative = $f_{\vec{u}}(a,b) = \vec{\nabla} f \cdot \vec{u}$, where $\vec{\nabla} f = f_x \hat{i} + f_y \hat{j}$ and $\vec{u} = u_1 \hat{i} + u_2 \hat{j}$.
  - f_x and f_y are "directional derivatives"
  - Need 3 things to find a directional derivative:
    1. The function, $f(x,y)$.
    2. The point, (a,b).
    3. The direction of interest, $\vec{u}$.
  - $\vec{\nabla} f$ is determined by the partial derivatives of the function at the point – it does not change once you have chosen the point.
  - Once you have chosen the (a,b) point the only thing you get to choose is the direction you move, $\vec{u}$.
- Properties of the Directional Derivative
  - For a given point (a,b),of all the directions you can choose to move, in which direction is the greatest rate of change?
    - $\vec{\nabla} f$ points in the direction (since it is a vector)of Greatest Rate of Increase in *f*
    - $-\vec{\nabla} f$ points in the direction of Greatest Rate of Decrease in *f*.
    - $\left\| \vec{\nabla} f \right\|$ gives the rate (since it is a scalar) of change of *f* in the direction of Greatest Rate of Increase, or the greatest rate of increase.
    - Example: The temperature, in F°, at a location in the xy plane, measured in cm, is given by $f(x,y) = 150 - x^2 - y^2$.
      a. If you are located at the point (3,7), in which direction is the temperature increasing fastest?
      b. How fast is the temperature changing in that direction?
    - Computer Demo
  - $\vec{\nabla} f$ is perpendicular to the contour of *f* at the point *(a,b).*
  - $\left\| \vec{\nabla} f \right\|$ is large when the contours are close together and small when the contours are far apart.
    - Example: Draw in the direction of $\vec{\nabla} f$ at the point (1,1) on the contour diagram. Also indicate where $\left\| \vec{\nabla} f \right\|$ would be larger in the contour plot.

You Try It

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- Recap Properties of $\vec{\nabla} f$ at a given point (a,b).

- $\vec{\nabla} f$ points in the <u>direction</u> of Greatest Rate of Increase in f
- $-\vec{\nabla} f$ points in the <u>direction</u> of Greatest Rate of Decrease in f
- $\left\|\vec{\nabla} f\right\|$ gives the greatest <u>rate</u> increase.
- $\vec{\nabla} f$ is perpendicular to the contour, or level curve, of f at the point (a,b)
- $\left\|\vec{\nabla} f\right\|$ is large when the contours(or level curves) are close together and small when the contours are far apart.