Day 11
Differentials

- Review HW problem 14.3 #17
- The tangent approximation of the function is
  - $f(x,y) \approx z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b)$
    $f(x,y) - f(a,b) \approx f_x(a,b)(x-a) + f_y(a,b)(y-b)$
    $\Delta f \approx f_x(a,b)\Delta x + f_y(a,b)\Delta y$ Differential (we'll use this idea later today!)
    as $\Delta x \to 0$ and $\Delta y \to 0$
    $$df = f_x(a,b)dx + f_y(a,b)dy$$

You Try It
Section 14.3 #13 Answer in Text

Directional Derivatives

- Again, recall from Calculus I……..
  - $f'(x) \approx \dfrac{f(x+h)-f(x)}{h} = \dfrac{\Delta f}{h}$
    $f'(x) = \lim\limits_{h\to 0} \dfrac{\Delta f}{h}$

  We'll use similar notation next.
- In Calculus III………
  - **Directional Derivative:** Instantaneous Rate of Change in the direction given by the **unit** vector, $\vec{u}$, from the point in the xy-plane, (a,b). The symbol is $f_{\vec{u}}(a,b)$.
  - $f_{\vec{u}}(a,b) = f_x(a,b)u_1 + f_y(a,b)u_2$, where $\vec{u} = u_1\hat{i} + u_2\hat{j}$
  - Example: Find $f_{\vec{u}}(1,2)$ for the function $f(x,y) = x^2y^5$ in the direction given by $\vec{v} = 3\hat{i} + 4\hat{j}$.
  - This can be seen as the dot product between two vectors: the unit vector $\vec{u}$ and another vector consisting of the partial derivatives. This other vector is called the **gradient vector** or $\vec{\nabla} f = f_x\hat{i} + f_y\hat{j}$
    - Example: Find $\vec{\nabla} f$ where $f(x,y) = x^2y^5$ at the point (1,2).

You Try It
Section 14.4 #17. Answer in text.

  - Therefore the **directional derivative**, or rate of change in the function in a given direction, is the dot product of the unit vector giving the direction and the vector $\vec{\nabla} f$ (read Del f) or $\overrightarrow{grad}\ f$.
  - **Directional Derivative** $= f_{\vec{u}}(a,b) = \vec{\nabla} f \cdot \vec{u}$
  - Example: The temperature, in F°, at a location in the xy plane, measured in cm, is given by $f(x,y) = 150 - x^2 - y^2$. How fast is the

temperature changing at the point (3,7) in the direction $\vec{v} = 3\hat{i} - 4\hat{j}$?
Interpret the answer in the context of the problem.

You Try It

An ant is at the point (3,4) on a metal plate and heads toward the point (5,0). Use the function for temperature given above. What is the instantaneous rate of change in temperature in that direction? Interpret the answer in the context of the problem. Hint: Make a unit vector from the point (3,4) to the point (5,0) first. Video Solution.