Day 10

Local Linearity and Tangent Planes

- Local Linearity
  - Recall from Calculus I………
    - Zoom in on the graph of $y(x)=e^{-0.2x}\sin(4x)$ at a particular point to see that as you get closer to the point the graph appears linear.
    - The derivative at that point will be the slope of the tangent line at that point.
    - The tangent line at that point will approximate the function y(x) well close to that point.
    - In fact, close to that point the y - value on the function, $y(x)=e^{-0.2x}\sin(4x)$, will be approximately equal to the y-value on the tangent line. The further away you get from the point, potentially, the worse the tangent line will approximate the function value.
  - In Calculus III……
    - Linear in 3D means a plane.
    - Zoom in on the graph of $z(x,y)=x^2+y^3$ near the point (2,1,5).
    - Zoom in on the contour diagram near the point.
    - At the point (a,b), the slope in the x-direction, m, will be $f_x(a,b)$and the slope in the y-direction, n, will be $f_y(a,b)$.
    - As in 2 space, the plane will approximate the function well near the point.
- Tangent Planes
  - Recall from Calculus I…….
    - Equation of tangent line
      - $$y-y_1=m(x-x_1)$$ $$y-f(x_1)=f'(x)(x-x_1)$$ $$y=f'(x)(x-x_1)+f(x_1)$$

  You Try It

  Find the equation of the tangent line to the function $y(x)=e^{-0.2x}\sin(4x)$ at x = 1.825 Answer: y = 1.343x-1.86

  - In Calculus III……..
    - Equation of *tangent plane*
      - $$z=m(x-a)+n(y-b)+f(a,b)$$ $$z=f_x(a,b)(x-a)+f_y(a,b)(y-b)+f(a,b)$$
      - This is equation of the tangent plane at the pint (a,b).
      - As we said when discussing local linearity, the function value f(x,y) will be approximately equal to the z on the line near the point (a,b). Or, $f(x,y)\approx z$.

- Example: Find the equation of the tangent plane to $z(x,y) = x^2 + y^3$ at the point (2,1).

You Try It

Section 14.3 #3 Answer in text.